QUANTUM FIELD THEORY FOR PHILOSOPHERS

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'The quantum theory of fields is the contemporary locus of metaphysical research'

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ABSTRACT:

The metaphysical commitments of quantum field theory are examined. A thesis of *underdetermination* as between field and particle approaches to the elementary particles is argued for but only if a disputed notion of transcendental individuality is admitted. The superiority of the field approach is further emphasized in the context of heuristics.

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1. Introduction

It is now generally recognized that Quantum Field Theory (QFT), after many vicissitudes and a period of almost total neglect, has emerged during the last ten years as a triumphant framework in which to do elementary particle physics.

If philosophers are prepared to use current physical theory as a guide to resolving metaphysical questions (and that is a big 'If'), then QFT is what philosophers should all be studying. But the subject is notoriously recondite, and philosophers are busy people, so the objective of the present paper is to cut through all technicalities and try to tell in as straightforward a way as possible just what QFT has to say about 'reality'. Of course there will be many places where questions will have to remain open, sometimes because the technical answers are not known (to the author!), but more importantly because the conceptual foundations of QFT are genuinely obscure in a number of respects. So there will be question marks - indications for further discussion and analysis.

The plan of the paper is as follows:

In Section 2 I consider the classical concept of field. Here the literature is superabundant. I am just going to summarize, emphasizing those aspects of particular importance in QFT. In Section 3 I explain the basic ideas of relativistic QFT. One feature that emerges is the creation and destruction of particles (quanta). In Section 4 I look briefly at classical physics again to see how revolutionary this idea really is. One claim often made for QFT is that it resolves the problem of wave-particle duality in quantum mechanics. That is not really right and in Section 5 I explain why. In Section 6 I look at the distinction between 'matter' fields and 'force' fields and link this with a discussion of the unification of physics widely publicized as a major breakthrough in QFT. Then in Section 7 I consider the claim that elementary particles are non-individuals, and the issue of *underdetermination* of field versus particle theories is explored in the context of parafield theory. In Section 8 I consider the interpretation of the vacuum, and the rôle of

so-called virtual particles in QFT. Finally in the concluding Section I introduce a new category of 'ephemerals' which are intended to contrast with the more familiar 'continuants' as a more appropriate description for the elementary particles, and put forward a positive recommendation for field approaches to quantum mechanics in the area of heuristics.

It should be stressed that this paper is not concerned with the interpretative problems of quantum mechanics as such, but only with the additional problems posed by QFT.

2. The Classical Concept of Field

In classical physics it is usual to distinguish particle theories from field theories. The distinction arises in various ways. Firstly there is the contrast between different approaches to the theory of matter. Secondly there is the question of the nature of forces or interactions between 'pieces' of matter. The field theory of such interactions is opposed to 'action-at-a-distance' doctrines that are sometimes associated with 'pure' particle theories. There is a lot of 'fine-structure' in these very crude classifications, some of which will emerge in the ensuing discussion.

We start with a rough sort of definition:

A field theory in physics is a theory which associates certain properties with every point of space and time.

Examples: Electromagnetic field theory associates electric and magnetic forces with space-time points. Eulerian hydrodynamics associates a velocity with each space-time point which represents the velocity of that element of fluid which occupies the indicated position at the indicated time, and so on.

This definition makes it look at though space-time points ar the primary individuals of the theory. The fields are then properties of these spatio-temporal individuals. By contrast consider particle theories:

A particle theory in physics is a theory which attributes to certain individuals (the particles) a variety of properties.

These properties will include spatio-temporal locations. Notice

carefully the different rôle played by spatio-temporal locations in our definitions of field theories and particle theories. In field theories they are individuals associated with properties (the field quantities), in particle theories they are properties associated with individuals (the particles).

But what do we mean by individuals in this discussion? In classical physics the answer normally assumed was a naive 'substance'approach. An individual is an entity which is the 'bearer' of properties - if you strip away all the properties you are left with a substratum indicated by a proper name that picks out that individual as itself at one particular time and reidentifies it 'transcendentally' as the same individual at other times in its history. Individuation that transcends the properties of an entity in this way we shall refer to as transcendental individuality, TI for short[1]. The view of individuality I have sketched, more or less Locke's view in the case of atoms, is very controversial and would not be accepted in this crude form by many modern philosophers. It is however the way classical physics normally treats individuals[2]. But as we shall see later this is the only way one can treat individuals in quantum mechanics, (assuming the completeness of the quantum-mechanical state description) and this will be used as one line of argument in favour of a field approach to the interpretation of quantum mechanics.

Let us consider the classical particle mechanics of equal point masses. At first sight this looks like a paradigm example of a particle theory. Properties associated with the particles include their positions and momenta, i.e. their location in 6-dimensional 1-particle phase space. For N particles, distinguished by subscripts which play the role of proper names, the coordinates and momenta $\underline{q}_i$ $\underline{p}_i$, i = 1...N, of all the particles can be assembled into a 6N-dimensional phase space for representing the dynamical state of the complete N-particle system. But it was realized at least as early as Newton[3] that particle mechanics can be represented as a field theory. Instead of a particle as a primary individual we introduce a dichotomic field $\psi(\underline{q}, t)$ which can take on just two values {yes, no}. 'Yes' indicates that a particle is present at that space-time location. 'No' indicates that a particle is not present. For Newton 'particle' in this field formulation is identified with the quality or property of 'impenetrability'. To say 'Yes' to the question 'Is there a particle at the location $(\underline{q}, t)$ is just to

say the point (x, t) has associated with it the quality of impenetrability, an 'infinite' repulsive force field if you like.

The motion of a particle is represented by the motion of a 'spike' in the dichotomic field.

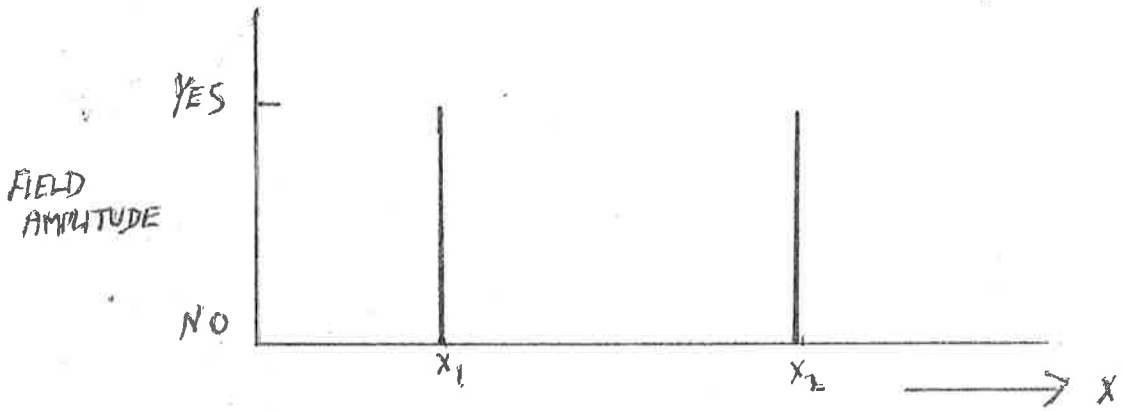

Fig.1.

Consider for simplicity motion in one dimension, labelled x (see Fig. 1). A particle at time t_1 is at location x_1 and moves at time t_2 to location x_2. This can be represented as a change in field configuration from the spike at x_1 to the spike at x_2. A property, impenetrability, has been passed on from location x_1 at t_1 to location x_2 at t_2. This is a typical field description, as opposed to the particle description, an individual (particle) has moved from x_1 at t_1 to x_2 at t_2.

There is no 'observable' difference between the field and particle description. We have here an excellent example of the underdetermination of theories by empirical data. Ontologically speaking the particle and field descriptions of what is going on when a particle moves from x_1 to x_2 is totally different, but no experiment will ever decide which is correct. Responses to underdetermination vary from Arrogance (there is no matter of fact at stake) to Ignorance (there are matters of fact which can never be decided by observation)[4]. In Section 7 below we shall find that underdetermination also exists between quantum field and particle theories. But let us return briefly to classical fields. Field theories of matter were prominent in the 18th century[5] (Hutton, Priestley etc). The primacy of force field over particle

source for the field was famously expounded by Faraday[6]. So already our discussion of field theories of matter has led us to field theories of force, or interaction between elements of matter. The dualistic approach here is to have two categories of entity, particles (with TI) and forces (action-at-a-distance) between them. But could not forces be reduced to particles? Well effluvial theories, forces due to the impact of streams of particles, were popular in the 17th and 18th centuries[7]. But another way was, not to transmit action by the actual transport of particles, but to consider a medium (perhaps also ultimately particulate) between the particles in which action between the particles was mediated by passing a state of stress or motion from one location in the medium to another. The individual particles of the medium did not move right across from one material particle to another, just a state of the medium was propagated. We may call such theories wave theories of interaction in a very general sense of 'wave'. This was the general attitude of 19th century ether theories, at any rate the earlier ones[8]. But ether theories developed in various directions. Firstly why have material particles and ether? Could we not have a monistic theory in which matter was reduced to some particular local state of the ether - like Thomson's vortex atom theory[9]? The most sophisticated version of this approach was due to Larmor[10]. But dualism was revived in a different way by Lorentz[11]. Now there were charged particles of matter and an ether which was not conceived of in a mechanical way. The ether was just the Maxwell fields of electrical and magnetic force. But although the Lorentzian ether was not mechanical, it was conceived of, in a certain sense, as substantial. This was mainly due to the fact the field carried with it energy, whose density and flux were related in Poynting's theorem. To say that the electromagnetic field was substantial was to say no more and no less than that it carried (was associated with) energy (and indeed momentum). Energy as a new ontological category had emerged as a major theme in 19th century physics[12]. At first, for Helmholtz, all energy was ultimately mechanical, but this reductionist approach

was relaxed by the energeticist of the latter part of the 19th century. Then The dualism of Lorentz got replaced by a new monistic electromagnetic theory of matter (Abraham, Wien etc). Perhaps matter was just a local concentration of electromagnetic field (Mie , Born and Infeld). But now Einstein and relativity had intervened. At first regarded as reactionary in the context of the electromagnetic theory of matter, it gave extra support to the substantial nature of energy, now associated via $E = mc^2$ with inertial, and via the Equivalence Principle with gravitational, mass. But general relativity introduced a new field into physics, the metric tensor field which 'explained' gravitational force. There emerged for Einstein and his collaborators the vision of a unified field theory, in which the electromagnetic field as well as the gravitational could be given geometrical significance, and matter, instead of being associated with singularities in this generalized field, points at what the field equations did not apply, would again be identified with local concentrations of this field. An offshoot of the Einsteinian approach was the geometrodynamics of Wheeler[13], conceived first as a unified field theory (in the sense of Rainich)but later identified with remarkable properties of the coupled Einstein-Maxwell equations (theory of geons etc).

With this brief background of classical field theories in mind we want now to explain quantum field theory

3. Quantum Field Theory

There are two main approaches to QFT. Firstly we consider a classical field such as the Maxwell field and formulate it as a classical Hamiltonian theory of a system with an infinite number of degrees of freedom. The magnitude of each field component at every point of space is regarded as a generalized coordinate, the totality of such magnitudes specifying the state of the system in configuration space. To get a generalized phase space we need to associate also a 'momentum' with every point of space

canonically conjugate to the field amplitude. The physical energy and momentum of the field are then functionals of these 'configuration' and 'momentum' fields. To turn this classical Hamiltonian field theory into QFT we replace the 'configuration' and 'momentum' fields by operator fields (i.e. each point in space and time is associated with an operator in an appropriate Hilbert space - this is what we mean by an operator - valued field or just an operator field for short). The commutation properties of these operators must be specified and then we are ready to start doing quantum field theory as a natural generalization of the familiar quantum mechanics of systems like harmonic oscillators and hydrogen atoms with a finite number of degrees of freedom. How are we going to specify the commutation properties of our operator fields? Well the simplest suggestion is just to adapt the usual canonical commutation relation $[Q,P] = i\hbar$ where Q is a position operator, P a canonically conjugate momentum operator and $\hbar$ is Planck's constant divided by 2π. If $Q(\underline{x},t)$ is a configuration operator field and $P(\underline{x},t)$ is the canonically conjugate momentum operator field, then we impose the equal-time commutation relations

$$\left.\begin{aligned} \left[Q(\underline{x},t), P(\underline{x}',t)\right] &= i\hbar\,\delta(\underline{x}-\underline{x}') \\ \left[Q(\underline{x},t), Q(\underline{x}',t)\right] &= \left[P(\underline{x},t), P(\underline{x}',t)\right] = 0 \end{aligned}\right\} \qquad (1)$$

We are working in the Heisenberg picture for time-development in which the state vector is time-independent and the operators are time-dependent. The equations of motion for the operator fields are then just the classical equations of motion for the field, which can be solved so as to compute commutation properties at different times in terms of the commutation properties at equal times.

The Maxwell field suffers from some technical complications when one tries to implement the programme sketched above, which need not concern us. To fix our ideas let us treat briefly[14] the case of a real scalar field ψ satisfying the Lorentz-invariant Klein-Gordon equation

$$\left(\nabla^2 - 1/c^2 \frac{\partial^2}{\partial t^2} + \mu^2\right)\psi = 0 \qquad (2)$$

where c is the velocity of light and μ is a real parameter whose significance will become apparent in a moment.

Because we are quantizing a relativistic wave equation we are doing relativistic quantum field theory.

ψ is the 'configuration' field, the canonically conjugate 'momentum' field for this case turns out to be

$$\Pi = \frac{\partial \psi}{\partial t} \qquad (3)$$

The Hamiltonian (total energy) of the field is

$$H = \int H \, d^3\underline{x} \qquad (4)$$

where the Hamiltonian density H is given by

$$H = \frac{1}{2}\left\{\Pi^2 + c^2 |\nabla\psi|^2 + c^2\mu^2\psi^2\right\} \qquad (5)$$

Instead of specifying the 'configuration' field by giving the amplitude of the field at each point of space we can equally well specify the field by giving the time-dependent Fourier transform of the field amplitude.

Thus we write

$$\psi(\underline{x},t) = V^{-\frac{1}{2}} \sum_{\underline{k}} q_{\underline{k}}(t) e^{i\underline{k}\cdot\underline{x}} \qquad (6)$$

Similarly

$$\Pi(\underline{x},t) = V^{-\frac{1}{2}} \sum_{\underline{k}} p_{\underline{k}}(t) e^{-i\underline{k}\cdot\underline{x}} \qquad (7)$$

where we have assumed periodic boundary conditions for the fields on the faces of a very large cube of volume V. $\sum_{\underline{k}}$ denotes summation over the discrete Fourier components allowed by the boundary conditions.

In order that ψ and Π be real (Hermitian) fields we impose the conditions

$$\left.\begin{aligned} q_{-\underline{k}} &= q_{\underline{k}}^{\dagger} \\ p_{-\underline{k}} &= p_{\underline{k}}^{\dagger} \end{aligned}\right\} \tag{8}$$

The equal-time commutation relations (1) now imply

$$\left.\begin{aligned} \left[q_{\underline{k}}(t), q_{\underline{k}'}^{\dagger}(t)\right] &= i\hbar\,\delta_{\underline{k},\underline{k}'} \\ \left[q_{\underline{k}}(t), q_{\underline{k}'}(t)\right] &= \left[p_{\underline{k}}(t), p_{\underline{k}'}(t)\right] = 0 \end{aligned}\right\} \tag{9}$$

The Hamiltonian reduces to

$$\mathcal{H} = \frac{1}{2}\sum_{\underline{k}} \left(p_{\underline{k}}^{\dagger} p_{\underline{k}} + \omega_{\underline{k}}^{2} q_{\underline{k}}^{\dagger} q_{\underline{k}}\right) \tag{10}$$

where $$\omega_{\underline{k}} = c\sqrt{\mu^2 + \underline{k}^2} \tag{11}$$

What we have done is to introduce a description of the field in terms of the excitations of various modes indexed by the label $\underline{k}$. The total energy is a sum over the decoupled contributions from each mode.

But the summand in (10) is quite similar to the familiar Hamiltonian for the 3-dimensional harmonic oscillator[15]. Effectively we have reduced QFT to the quantum mechanics of an assemblage of harmonic oscillators!

To find the eigenvalues of $\mathcal{H}$ we can easily adapt the well-known results for the harmonic oscillator.

We introduce new time-independent operators $a_{\underline{k}}, a_{\underline{k}}^{\dagger}$ via the equatio

$$\left.\begin{aligned} q_{\underline{k}}(t) &= \sqrt{\frac{\hbar}{2\omega_{\underline{k}}}}\left(a_{\underline{k}} e^{-i\omega_{\underline{k}} t} + a_{-\underline{k}}^{\dagger} e^{i\omega_{\underline{k}} t}\right) \\ p_{\underline{k}}(t) &= i\sqrt{\frac{\hbar\omega_{\underline{k}}}{2}}\left(a_{\underline{k}}^{\dagger} e^{i\omega_{\underline{k}} t} - a_{-\underline{k}} e^{-i\omega_{\underline{k}} t}\right) \end{aligned}\right\} \tag{12}$$

This ensures that the field satisfies the equation of motion (2) (which must also be satisfied by Π in virtue of (3)) and the reality conditions (8).

The commutation relations (9) require

$$\left.\begin{aligned} \left[a_{\underline{k}}, a_{\underline{k}'}^{\dagger}\right] &= \delta_{\underline{k},\underline{k}'} \\ \left[a_{\underline{k}}, a_{\underline{k}'}\right] &= \left[a_{\underline{k}}^{\dagger}, a_{\underline{k}'}^{\dagger}\right] = 0 \end{aligned}\right\} \tag{13}$$

If we define

$$N_{\underline{k}} = a_{\underline{k}}^{\dagger} a_{\underline{k}} \tag{14}$$

then the eigenvalues of $N_{\underline{k}}$ can easily be shown to be just the non-negative integers 0, 1, 2...

Furthermore we have

$$H = \hbar \sum_{\underline{k}} \left(N_{\underline{k}} + \tfrac{1}{2} \right) \omega_{\underline{k}} \qquad (15)$$

It turns out that the total (physical) momentum of the field is

$$\underline{P} = \hbar \sum_{\underline{k}} N_{\underline{k}} \cdot \underline{k} \qquad (16)$$

So if we choose a representation in which all the $N_{\underline{k}}$ are diagonal with eigenvalues $n_{\underline{k}}$, then H and $\underline{P}$ will be simultaneously diagonal with eigenvalues

$$E = \sum_{\underline{k}} \left(n_{\underline{k}} + \tfrac{1}{2} \right) \left(\hbar \omega_{\underline{k}} \right) \qquad (17)$$

and

$$\underline{P} = \sum_{\underline{k}} n_{\underline{k}} \cdot \left(\hbar \underline{k} \right) \qquad (18)$$

The representation with all the $N_{\underline{k}}$ diagonal we shall refer to as the <u>particle representation</u> of the field since it corresponds to the energy and momentum spectrum of a collection of particles $n_{\underline{k}}$ of which have momentum $\hbar \underline{k}$ and energy $\hbar \omega_{\underline{k}}$, if for the moment we ignore the (infinite!) constant term provided by the factor $\frac{1}{2}$ in Eq.(17) for E. It is called the <u>zero-point energy</u> of the field.

Remembering the definition of $\omega_{\underline{k}}$ given in (11) we infer that the significance of the parameter μ in Eq.(2) is just that $\hbar\mu/c$ is the rest-mass of the 'particles' associated with the field on quantization.

So starting with a classical field and quantizing it we have arrived at a theory that <u>in the particle representation</u> looks just like a theory of (non-interacting) particles (hence the terminology). We shall term the number $n_{\underline{k}}$ the <u>excitation number</u> of the $\underline{k}$ -mode. We then have the simple basic result of QFT:

> The number of particles present with momentum $\hbar \underline{k}$ and energy $\hbar \omega_{\underline{k}}$ in the <u>particle representation</u> of the theory is just the excitation number of the $\underline{k}$ -mode.

The significance of the operators $a_{\underline{k}}$ and $a^{\dagger}_{\underline{k}}$ introduced in (12) is that $a_{\underline{k}}$ is an annihilation operator for particles of momentum $\hbar\underline{k}$ in the sense that

$$a_{\underline{k}} \, | n_{\underline{k}} \rangle = \sqrt{n_{\underline{k}}} \, | n_{\underline{k}} - 1 \rangle \qquad (19)$$

while $a^{\dagger}_{\underline{k}}$ is a creation operator for particles of momentum $\hbar\underline{k}$ in the sense that

$$a^{\dagger}_{\underline{k}} \, | n_{\underline{k}} \rangle = \sqrt{n_{\underline{k}} + 1} \, | n_{\underline{k}} + 1 \rangle \qquad (20)$$

In a sense we can say that starting with a field and quantizing it we have ended up with a particle aspect for the field.

Now let us turn to a second quite distinct way of approaching QFT. We begin with the non-interacting N-particle Schrödinger equation and impose on the solution the condition of symmetry under particle label permutations. So we are looking at a particle theory of bosons, defined just as particles whose state vectors are totally symmetric under particle label permutations. In view of this symmetry property, the state of an N-particle system, formed by symmetrization from the tensor product state assigning each particle to a particular 1-particle state, is totally specified by giving the number of particles n_i in the i-th 1-particle state $|u_i\rangle$. Suppose $|u_i\rangle$ is an eigenstate of the 1-particle Hamiltonian with eigenvalue E_i. Then the energy of such a symmetrized state is given simply by

$$E = \sum_i n_i E_i \qquad (21)$$

But the formula for the energy of an assemblage of distinguishable harmonic oscillators of angular frequency ω_i given by $\omega_i = E_i/\hbar$ and excitation number n_i for the i-th harmonic oscillator is just

$$E = \sum_i \left(n_i + \tfrac{1}{2} \right) E_i \qquad (22)$$

If we re-set the zero from which we measure energy by substracting off the zero-point energy in (22) we can effectively identify the

two energy-formulae (21) and (22). But the result (22) is just what we would get by taking the 1-particle Schrödinger equation and subjecting it to a 'second quantization' by treating it as a classical field and following the quantization procedure we employed for the case of the Klein-Gordon field. In other words starting with the N-particle Schrödinger equation we can reinterpret our results as a quantum field theory in our first sense of the 1-particle Schrödinger equation, interpreted as a classical field equation.

For simplicity we have introduced the approach in a nonrelativistic context. Relativistic generalizations are considered in detail by Schweber (1961).

This method of reinterpreting a particle theory with symmetric wave functions as a quantized field theory we shall refer to as the second-quantization technique. In the way we have introduced it, it is no more than that, just a way of expressing a particle theory, and quite different from the first approach, which we shall refer to as field quantization, in which a 'real' field was subjected to a canonical quantization procedure.
Before commenting further on this situation we notice that the second-quantized version of the N-particle Schrödinger equation is actually more general than the latter. The latter is subject to the constraint

$$\sum_i n_i = N \tag{23}$$

But in the second-quantized version the n_i can be arbitrary numbers. The second-quantized version really operates with the direct sum of Hilbert spaces for the no-particle, 1-particle, 2-particle, etc problem. This is known as Fock space $\mathcal{F}$.

$$\mathcal{F} = K_0 \oplus K_1 \oplus K_2 \cdots \oplus K_N \oplus \cdots \tag{24}$$

where K_N is the Hilbert space for the symmetrized N-particle problem. K_0 comprises the vacuum state, where the excitation numbers of all the modes are zero.

If $a_i^\dagger$, a_i are the creation and annihilation operators associated with the state $|u_i\rangle$ then we can construct the quantized field

$$\Psi(\underline{r},t) = \sum_i a_i . u_i(\underline{r}) e^{-i/\hbar . E_i t} \qquad (25)$$

and its Hermitean conjugate

$$\Psi^\dagger(\underline{r},t) = \sum_i a_i^\dagger u_i^*(\underline{r}) e^{i/\hbar . E_i t} \qquad (26)$$

where $u_i(\underline{r})$ is the wave function associated with the state $|u_i\rangle$. $\Psi^\dagger$ and Ψ can be thought of as operators which create or destroy particles in states localized at the point $\underline{r}$ at time t.

The energy for the field is given by

$$\mathcal{H} = \int \Psi^\dagger . H(\underline{r},\underline{p}) . \Psi \, d^3\underline{r} \qquad (27)$$

where $H(\underline{r},\underline{p})$ is the 1-particle Hamiltonian with $\underline{p}$ interpreted as $-i\hbar\nabla$ in the usual way.

$\mathcal{H}$ is the Hamiltonian operator for the quantized field and it is important to notice that if we revert to the Schrödinger picture for the moment, we still have a time-dependent Schrödinger equation for the state of the quantized field expressed as

$$i\hbar \frac{\partial |\Phi\rangle}{\partial t} = \mathcal{H} . |\Phi\rangle \qquad (28)$$

where the state-vector $|\Phi\rangle$ is a vector in Fock space, and $\mathcal{H}$ is an operator in the same space. But since $\mathcal{H}$ involves a product of a destruction operator and a creation operator in the integral, it does not move a vector out of any particular component of $\mathcal{F}$ it happens to be in initially. Suppose we have a state vector lying in K_N describing the state of N particles. The action of an annihilation operator like Ψ is to move this vector into K_{N-1} but the subsequent action of a creation operates like $\Psi^\dagger$ moves us back into K_N again. The net result is a transition from one vector in K_N to another also in K_N. Schematically what is happening is the following. If an operator Q takes a vector in K_N into another vector in K_N,

then we can factorize Q in the form $Q = \xi\eta$ where η is an operator that takes us into K_{N-1}, an annihilation operator, and ξ is an operator that restores us to the final transformed vector in K_N, a creation operator. The situation is sketched in Fig. 2.

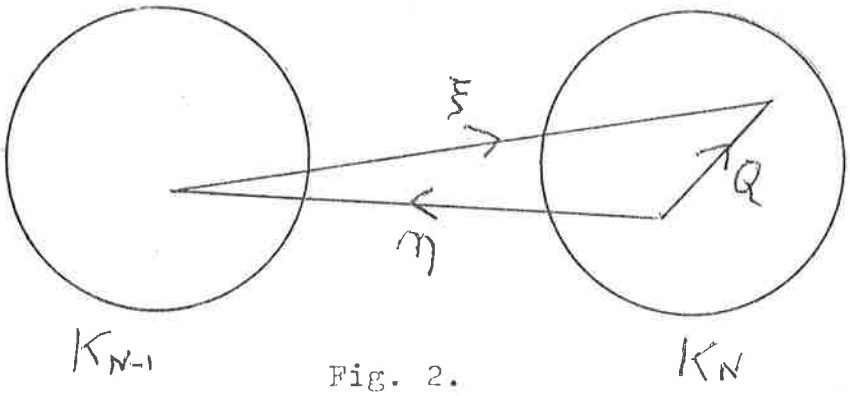

Fig. 2.

If $N \geq 1$, K_{N-1} would be the vacuum state Ko; a single particle is annihilated in one state, leaving us with the vacuum, and is then re-created in another state from the vacuum.

Clearly if we had an operator which was a linear combination of creation and destruction operators the effect on a vector in K_N would be to produce a linear combination of vectors in K_{N-1} and K_{N+1}, i.e. such an operator would describe the annihilation and creation of particles. Operators of this type occur when we consider interactions between fields and is the mechanism for explaining the emission and absorption of photons in radiation theory for example. Indeed the importance of QFT in particle physics arises in the first instance from the fact that QFT can very easily accommodate a theory of particle creation and decay (annihilation), characteristic features of most of the so-called elementary particles.

There is one more important point to notice. Consider a state in which one particle is present in each of N 1-particle states $|u_1\rangle, |u_2\rangle \ldots |u_N\rangle$.
This state can be written in the form

$$a_1^\dagger a_2^\dagger \ldots a_N^\dagger |\Phi_0\rangle \qquad (29)$$

where $|\Phi_0\rangle$ is the vacuum state. Notice that the suffices $1 \cdots N$ on the creation operators refer to the particle states, not of course to the particle labels - these do not appear at all in the second-quantized formulation. What about the fact that the original N-particle theory was a theory of bosons, with states symmetric under any permutation of particle labels? Well there is a 1 : 1 correspondence between particle label permutations and what we may call particle place permutations which act in the states (places) the particles are in, and the state (29) is symmetric under the latter sort of permutation because the creation operators all commute. In other words the commutativity of the creation operators is what corresponds in the second-quantized formulation to the label-permutation symmetry in the original N-particle formulation. Later we shall see that if we have different permutation properties imposed on the particle labels this will be reflected in different commutation properties of the creation operators.

Let us look back at what we have done so far. A natural reaction might be as follows. Field quantization and second-quantization both led to one sort of animal, a quantized field, but is the status of the quantized field the same in the two cases? In field quantization we started with a 'real' field. When we quantized it some particle properties attached themselves to the field. But there was no question of the field actually turning into a collection of particles, simply because there was never any mention of particle labels, i.e. there are no proper names for the particles and hence no TI, so they are not really particles at all, that was just a rather misleading way of describing the 'quanta' of energy and momentum associated with the field when we quantized it. As for second-quantization, can we not argue in reverse? The 1-particle Shrödinger field is not a 'real' field at all. The whole procedure is just a device for representing the mathematics of symmetric states of N-particle systems. The theory is a particle theory looked at in a rather Pickwickian way. This view of second-quantization is reinforced by reference to what happens if we consider a collection of

fermions, whose states are described by wave functions which are antisymmetric in the particle labels. The second-quantization formalism can be carried through for this case as well, but in specifying the commutation properties of creation and annihilation operators we now have to replace commutator brackets by anticommutato brackets. More specifically we now have

$$\left.\begin{aligned} \{a_i, a_j^\dagger\} &= S_{ij} \\ \{a_i, a_j\} = \{a_i^\dagger, a_j^\dagger\} &= 0 \end{aligned}\right\} \qquad (30)$$

where $\{\ \}$ signifies the anticommutator bracket specified by $\{A,B\} = AB + BA$ for any pair of operators A and B. In other words when we second-quantize the 1-particle Schrödinger equation we now have to use anticommutator brackets rather than commutator brackets to replace the Poisson brackets in the 'classical' field theory. But what are we to make of this version of 'second-quantization'? Is it properly a field quantization at all?

I want now to reply to this attempt at a categorical distinction between intrinsically field and particle theories[16] in the following way:

1) The fact that in our field-quantization example we considered a real Klein-Gordon field and the Schrödinger field is essentially complex has no bearing on the question whether the Klein-Gordon field is 'real' (in another sense of 'real'!) and the Schrödinger field 'unreal'. The original introduction of the Klein-Gordon equation was as a relativistic generalization of the Schrödinger equation, i.e. it was a complex field But a complex field can be considered as an ordered pair of real fields. In relativistic QFT complex fields are associated with charged particles - we need the two 'components' of the field to represent particles of opposite charge. Another way of expressing this is that relativistic QFT associates with every particle on antiparticle, with the same mass and spin, but carrying opposite additive quantum numbers like electric charge. It should be stressed that 'antiparticles',

in modern formulations of QFT, have nothing to do with negative energy states of 'particles'. It is true they were originally introduced by Dirac as 'holes' in a completely filled 'sea' of negative energy states, but there is no need to regard them in this way, and modern QFT makes no reference to hole theory[17].

2) A ~~The~~ one sort of classical limit ~~of~~ for quantized the Klein-Gordon or Maxwell fields is undoubtedly a field theory. This is due to quantization with commutator brackets which allow any positive integer (or zero) as eigenvalue for the number operator $N_{\underline{k}}$. The classical limit has 'intense' fields with large excitation numbers for the various modes, so quantal fluctuations in these excitation numbers can be ignored. Similarly the classical limit of fields quantized with anticommutator brackets, such as the second-quantized Schrödinger field of the electron, is a particle theory. This is because the excitation number for fermion fields is restricted to the values 0 or 1 (this just expresses the Pauli Exclusion Principle for such theories). Hence there is no classical field limit of 'intense' fields - the field aspect remains always quantal. But because two theories have different sorts of classical limit should not invite us to treat them as fundamentally distinct when we do not proceed to the limit. The idea that QFT starts from a classical theory (field or particle) and proceeds by 'quantizing' it is really an historical accident of how quantum theory actually developed. We shall see in a moment that there are other, more fundamental, ways of introducing quantization that do not depend on the canonical Hamiltonian approach.

3) If the distinction between truly 'field' and 'particle' theories is not tied to the distinction between bosons and fermions could it be tied to the distinction between massless fields and massive fields? Massless fields are just fields like the Maxwell field whose quanta have zero rest-mass. The question being raised here is that the quanta associated with massless fields are often claimed not to be 'localizable'. Hence they cannot really be particles. The point is somewhat technical and I shall just sketch what is going on. We begin with the first-quantized theory.

The states of an elementary system should certainly support irreducible representations of the Poincaré group (inhomogeneous Lorentz group). These representations were classified in a fundamental paper by Wigner (1939). The representations of physical interest are characterized by two parameters, the rest-mass m, which can take on any real value and the spin S, which has integral or half-integral values, in appropriate units. The representations with m = 0 are actually quite different from those with m ≠ 0. The so-called 'little group' is non-compact (the Euclidean group in two dimensions whereas for m ≠ 0 it is the compact 3-dimensional rotation group). This is the basic distinction between massive and massless particles. Newton and Wigner (1949) laid down certain conditions which must be satisfied by a 'position' operator for a particle. In the case of the photon ($m = 0$, $S = 1$) these conditions could not be satisfied, and they concluded that there was no 'position' operator for the photon, i.e. no operator whose eigenvalues could be interpreted as the position of the photon. For neutrinos ($m = 0$, $S = 1/2$) the situation turned out to be more complicated. Wightman (1962) showed that for neutrinos that lacked a definite helicity a position operator could be constructed. Furthermore Jauch and Piron (1967) suggested that the Newton-Wigner conditions were perhaps overly restrictive. Subject to a new weaker set of conditions they showed that the photon was localizable! But even if a position operator does exist in a first-quantized theory what happens when we introduce second-quantization? A new difficulty now appears. If a particle is localized by measurement to within a volume of radius $\hbar/mc$, the resulting energy uncertainty will be sufficient to produce an additional particle, via the $E = mc^2$ relation. So the effective size of a particle of mass m in relativistic QFT is $\hbar/mc$, and this tends to infinity as $m \to 0$. It is again not very clear what bearing this has on regarding massless fields as not being really particle theories at all. They are particle theories in which the creation of additional particles is rather easy, at any rate if the particles have very low momenta, (this is the origin of the so-called infra-red catastrophe in QED, the unlimited production of soft photons accompanying all real

scattering transitions). But this circumstances of itself does not seem to prevent one regarding massless QFT as a particle theory at all.

4) Let me then sketch a point of view from which all types of relativistic field theory are perfectly well viewed as particle theories. We start with 1-particle states of definite momentum, mass, spin, and z-component of spin, which support the Wigner irreducible representation of the Poincaré group. These are often referred to as canonical states. We can easily introduce a Fock space with creation operators which produce such states by operating on the vacuum and corresponding annihilation operators in the usual way. Boson or fermion statistics are obtained by choosing commutator or anticommutator brackets for these operators. The canonical 1-particle states have complicated transformation properties under Lorentz transformations quite different from the corresponding properties of solutions of relativistic wave equations like those of Klein-Gordon, Dirac, or Maxwell, which transform under finite non-unitary representations of the Lorentz group. (More technically the canonical states transform under infinite-dimensional unitary representations). But we can easily introduce a change of basis vectors[18] such that fields contructed by (covariant) Fourier transformation from the creation and annihilation operators appropriate to the new basis do satisfy the simple transformation properties that characterize solutions of relativistic wave equations. Going back to the basis of canonical states enables us to express these 'fields' directly in terms of the creation and annihilation operators for the canonical states. But these 'fields' have not been constructed by quantizing a relativistic wave equation, and in general the fields need not satisfy any relativistic wave equation over and above the Klein-Gordon equation, which is always trivially satisfied by relativistic fields, that are associated on quantization with particles of unique rest-mass.

5) If all our quantized fields can be regarded as particle theories is there any point of view from which the quantized field itself can be regarded as having ontological significance?

The standard answer to this question is that the fields are the objects in terms of which a 'locality' or 'causality' condition can be imposed on the theory. What this means is as follows. Let $Q(x), Q'(x)$ be local observables, i.e. observable quantities constructed at the space-time point x from the field quantities specified at x[19]. Typical examples of local observables would be energy and momentum densities, current and charge densities and so on. The fields themselves may well not be included. In particular this will always be the case with fermion fields. The 'causality' condition is that

$$[Q(x), Q'(y)] = 0 \quad \text{if } (x-y)^2 < 0 \tag{31}$$

i.e. if x and y have space-like separation. The reason why this condition expresses a locality or causality condition is that if it were violated then a measurement of $Q(x)$ could instantaneously affect the statistics associated with measurements of $Q'(y)$[20]. If we impose the causality condition with $Q=Q'$ as a necessary condition on what will count as a local observable, then fermion fields are not themselves observable since they anticommute at space-like separation. Since boson fields commute at space-like separation they may be observables. Now if the field quantities commute or anticommute at space-like separation then local observables such as Q, Q' constructed as polynomials out of the fields will commute at space-like separation provided they contain an even number of fermion fields. This is certainly true for energy and current densities, etc. In other words a sufficient condition for causality is that the fields commute or anticommute at space-like intervals. We now quote a fundamental result of relativistic QFT:

Spin-Statistics Theorem:

> If fields commute at space-like intervals (boson fields) then the spin of the associated particles must be an integer. In this case the anticommutator does not vanish.
> If fields anticommute at space-like intervals (fermion fields) then the spin of the associated particles must be half-integral. In this case the commutator does not vanish.

Its real significance is to rule out quantization by anticommutator brackets for integral spin fields and quantization

by commutator brackets for half-integral spin fields. It is only if we restrict ourselves to these two quantization possibilities (commutation or anticommutation) that the causality condition imposes a unique quantization procedure. But is this restriction correct? To answer this question we must examine the justification for quantization procedures that are not tied to canonical quantization of a classical field, which gives us only the commutation option anyway. The problem was tackled in a classic paper by H.S. Green (1953) entitled 'A Generalized Method of Field Quantization'. In canonical quantization one starts with the commutation relations and then derives the equation of motion for the field via the relation

$$\frac{\partial \psi(\underline{x},t)}{\partial t} = i/\hbar \cdot [H, \psi(\underline{x},t)] \qquad (32)$$

where H is the total energy (Hamiltonian) for the field. Green's idea, actually borrowed from Wigner (1950), was to invert this sequence. A scheme of quantization would be considered satisfactory if it ensured the equation of motion, which demonstrates the total energy of the field as the infinitesimal generator of time-displacement, together with a corresponding interpretation of the total momentum of the field as infinitesimal generator of spatial displacement. Consider the case of a spin-zero field with Hamiltonian (15) which we write as

$$H = \frac{1}{2}\hbar \sum_{\underline{k}} \omega_{\underline{k}} \{a^{+}_{\underline{k}}, a_{\underline{k}}\} \qquad (33)$$

With the expansion (compare Eqs. (6) and (12))

$$\psi(\underline{x},t) = V^{-\frac{1}{2}} \sum_{\underline{k}} \sqrt{\frac{\hbar}{2\omega_{\underline{k}}}} \left(a_{\underline{k}} e^{i(\underline{k}\cdot\underline{x} - \omega_{\underline{k}} t)} + a^{+}_{\underline{k}} e^{-i(\underline{k}\cdot\underline{x} - \omega_{\underline{k}} t)} \right) \qquad (34)$$

the requirement (32) will be satisfied provided the relation

$$[a_{\underline{k}}, \{a^{+}_{\underline{\ell}}, a_{\underline{m}}\}] = 2\delta_{\underline{k},\underline{\ell}} \cdot a_{\underline{m}} \qquad (35)$$

together with the Hermitian conjugate of this relation holds for $\underline{\ell} = \underline{m}$.

Green assumes (35) to hold also for $\underline{\ell} \neq \underline{m}$ and supplements it with

$$\left[a_{\underline{k}}, \{ a_{\underline{\ell}}, a_{\underline{m}} \} \right] = 0 \qquad (36)$$

The trilinear commutation relations (35) and (36) are certainly satisfied by the usual boson commutation rule (13). But Green showed that there is a whole family of alternative commutation schemes, labelled by an integer p consistent with (35) and (36). The boson rule is obtained for p = 1. If we quantize with the p^{th} scheme one talks of a paraboson field of order p. Similarly Green considered generalizations of the fermion anticommutation rule for half-integral spin fields, and obtained the specification of what are now termed parafermion fields. The generalization of the spin-statistics theorem for the case of parafields (a generic term which includes parabosons and parafermions) was given by Dell' Antonio, Greenberg and Sudarshan (1964). The result is that paraboson commutation relations are excluded for half-integral spins, and parafermion commutation relations for integral spins. What emerges from this analysis is that the spin-statistics theorem does not uniquely prescribe the boson commutation rule for integral spin, and the fermion anticommutation rule for half-integral spin. All that we can say is that integral spin fields must be parabosons of arbitrary order while half-integral spin fields must be parafermions but again of arbitrary order. We shall return in Section 7 to discuss the relation between parafields and the corresponding particle theories. At this point we just want to emphasize that the causality condition on quantum fields does not have consequences that are quite as strong in delimiting the theory as is sometimes supposed[21].

4. Creation and Annihilation Operators in Classical Mechanics

We have seen that one of the most fundamental features of QFT is its ability to describe the creation and annihilation of 'particles'[22]. This looks particularly natural from the field quantization point of view. Particles are just quantized

excitations of particular modes of the field. Such excitations (unquantized) are perfectly familiar in classical field theories. The corresponding field configurations can come into being and go out of being without any question of creating or destroying (transcendental) individuals. Think of setting up a wave motion on a rope by shaking one end of it. From the Foch space particle point of view the creation and annihilation operators can be defined but they look a bit like a mathematical artifice. In this section I want to stress that Foch space as such has nothing to do with quantum mechanics. A Foch space description is equally well available in classical particle mechanics. This should not really surprise us since the commutations properties of the creation and annihilation operators do not involve $\hbar$, so letting $\hbar \to 0$ will not invalidate the description. But let us see how it would work out in practice. The time-development of the state of a classical particle takes place in phase space rather than Hilbert space. Restricting ourselves to a single particle and adapting the dichotomic field description introduced in Section 2, we represent the field as the limiting case of a continuous distribution $\rho(p_i, q_i, t)$ strongly peaked around the location of the particle in the phase space, where p_i and q_i are the momentum and position coordinates of the particle. The time-development of ρ is governed by Liouville's equation

$$\frac{\partial \rho}{\partial t} = \sum_i \left(\frac{\partial H}{\partial q_i} \frac{\partial \rho}{\partial p_i} - \frac{\partial H}{\partial p_i} \frac{\partial \rho}{\partial q_i} \right) \qquad (37)$$

where $H(p_i, q_i)$ is the Hamiltonian function.

So $$i \frac{\partial \rho}{\partial t} = L \rho \qquad (38)$$

where L is the so-called Liouville operator[23] defined by

$$L \underset{Df}{=} \sum_i \left(-i \frac{\partial H}{\partial p_i} \frac{\partial}{\partial q_i} + i \frac{\partial H}{\partial q_i} \frac{\partial}{\partial p_i} \right) \qquad (39)$$

Eq. (38) can be integrated formally to give

$$\rho(t) = e^{-iL(t-t_0)} \cdot \rho(t_0) \qquad (40)$$

showing the value $\rho(t)$ of ρ at time t as produced by the action of the operator $e^{-iL(t-t_0)}$ on the value $\rho(t_0)$ of ρ at time t_0.

But just as in the case of quantum-mechanical operators in the Fock space formulation we can formally factorize

$$e^{-iL(t-t_0)} = \xi \cdot \eta \qquad (41)$$

where η is an annihilation operator taking us from the initial field $\varphi(t_0)$ to the vacuum (defined by $\varphi = 0$) and ξ is an operator which recreates the particle from the vacuum in the final state $\varphi(t)$. Suppose we interpolate a denumerable infinity of instances $t_1, t_2 \ldots$ between t_0 and t. Then we can introduce the above description for each successive pair of instants (t_n, t_{n+1}). The description of time-development of the state of the particle in phase space consists now in claiming that the particle is being destroyed at every instant t_n and recreated at the successive instant t_{n+1}. In fact this mode of description has a nice parallel with the classic Arabic doctrine of the Kalâm espoused by the Mutakallemim[24]. Here time was discrete and 'accidents' did not persist across the time atoms but were continually destroyed and recreated, exactly in the way we have described. The continuing existence of the world depended on God's refraining from the decision not to recreate!

Finally we may remark that historically there was considerable resistance to accepting genuine particle creation and annihilation in physics. Even in the case of radiation Dirac in his (1927) originally interpreted the vacuum as a sink of unobservable photons, so radiation of a photon was not a genuine creation process, but just a transition from an unobservable to our observable state. This clearly has affinity with Dirac's later hole theory in which creation of a positron-electron pair was interpreted as the transition of an electron from a negative energy to a positive energy state. The whole-hearted adoption of genuine particle creation was really due to Fermi (1933) in his theory of β-decay.

5. Wave-Particle Duality

We have seen the equivalence between the N-particle Schrödinger equation and the 'second-quantized' 1-particle

Schrödinger equation. This equivalence was hailed by Dirac in his (1927) as a resolution of the wave-particle duality problem in quantum mechanics. Thus Dirac in discussing the interaction between atoms and electromagnetic waves refers to 'a complete harmony between the wave and light-quantum description of the interaction'[25].

According to the complementarity interpretation of wave-particle duality different phenomena, specified by the total experimental set-up, exhibited the wave and particle aspects of matter. But according to the above equivalence result any phenomenon could be described either by a quantized particle or a quantized wave theory. In this sense wave and particle theories were not mutually exclusive alternatives applicable to different complementary phenomena, but were united or better, transcended, by QFT. Reality was not a wave or a particle, but the single entity the quantum field. This kind of rhetoric conceals I believe a great deal of confusion. In the formalism of quantum mechanics complementarity is undoubtedly associated with the non-commutativity of the operators associated with certain pairs of observables. This raises a number of difficulties. Why is complementarity a two-place relation? It is after all easy to find more than two operators in Hilbert space which are pairwise noncommuting. Furthermore in the case of wave-particle duality, the relevant observables are usually regarded as position q and momentum p. But which of these observables has to have a sharp value if we are to talk of a wave or particle phenomenon? In the case of single-slit diffraction, a typical wave phenomenon, we are dealing with sharp q (in the idealization of an infinitely narrrow slit). But in other contexts the wave picture is often associated with sharp p (a definite wavelength) and the particle picture with sharp q (a definite position). As in all discussions of complementarity there is a good deal of 'uncertainty' in what we are talking about!

When we turn to QFT we have not of course eliminated complementarity, since we are still dealing with a quantum theory with plenty of noncommuting operators. In Section 2, in

discussing the real Klein-Gordon field, we introduced what we called the particle representation in which all the $N_{\underline{k}}$, the number operators for the various modes, were diagonal. But in this representation the field amplitude ψ is not sharp, since the creation and annihilation operators $a_{\underline{k}}^{\dagger}$ and $a_{\underline{k}}$ do not commute with $N_{\underline{k}}$. It seems reasonable to regard the representation in which ψ is diagonal as exemplifying the field (or wave) aspect of phenomena. Formally we can represent simultaneous eigenstates of $\psi(\underline{x}, t)$ for all $\underline{x}$ at some fixed time as a superposition of states with definite numbers of particles (excitation numbers) for the various modes. So the wave-particle duality problem can be regarded as reasserting itself in QFT in the sense that we cannot simultaneously ascribe sharp values to the particle number and the field amplitude. Alternatively we might regard a representation in which local observables such as the energy density H are simultaneously diagonal (for all $\underline{x}$) as the field picture. Of course H does not commute with ψ or with $N_{\underline{k}}$, so this is a good example of the situation referred to at the beginning of this section, where three sorts of operator are pairwise non commuting[26]. Once again, just as in ordinary quantum mechanics, there is some latitude in how we locate the wave-particle duality in the formalism, but to claim that the problem has been 'resolved' in QFT is clearly not right.

6. Matter Fields and Force Fields - Unification

In Section 1 we distinguished matter fields and force fields. How does this distinction look in QFT? So far we have talked exclusively about non-interacting fields. From the physics point of view this is all rather trivial - the interest begins when we consider interactions between fields[27]. Consider the case of quantum electrodynamics (QED). Here two fields are involved, the matter field describing the electrons and positrons via the Dirac equation and the force field specified by the Maxwell equation for the electromagnetic field. But the force field when quantized has a particle aspect (the photon). The photon is, as we have seen not quite a typical particle, in

view of its zero rest-mass, but insofar as it carries energy and momentum we would be inclined to regard it as substantial. So is not QED really a theory of the interaction of two sorts of particle? In turn it describes the interaction between two electrons, for example, as mediated by the exchange of photons, not 'real' photons but what are termed 'virtual' photons. This distinction will be explained in Section 8. The interaction is represented, in lowest non-vanishing order of perturbation theory, by the familiar Feynman diagram illustrated in Fig 3.

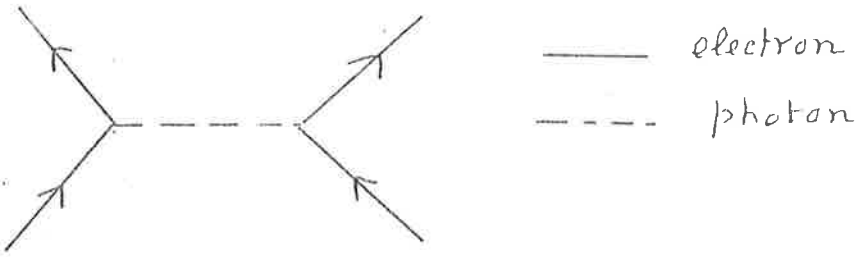

Fig.3

But we can equally well draw a Feynman diagram which illustrates the interaction of two photons as mediated by electrons and positrons. This is shown in Fig 4.

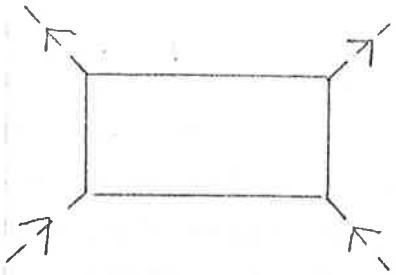

Fig.4

It is not at all clear which is the 'matter' particle and which is the 'force' particle. This lack of distinction between 'matter' particles and 'force' particles was particularly stressed in the so-called bootstrap approach to the elementary particles. For a detailed discussion of this approach and its relation to field theory see Redhead (1980a). In fact the emergence of the bootstrap approach in the 1960 s coincided with the virtual eclipse of field theory. But the fortunes of field theory revived in the 1970 s due to the success of what are known as gauge theories. Historically the first of the gauge theories was QED itself. The basic idea is as follows. The electron-positron field $\psi(x)$ satisfies the Dirac equation. The associated Lagrangian density is invariant under the global phase

transformation

$$\psi(x) \rightarrow \psi(x)e^{i\alpha} \qquad (42)$$

But suppose we impose a requirement of invariance under a variable (local) phase transformation

$$\psi(x) \rightarrow \psi(x)e^{i\alpha(x)} \qquad (43)$$

where $\alpha(x)$ is now an arbitrary space-time-dependent phase factor. The Dirac Lagrangian is no longer invariant and in order to restore invariance under this more restrictive class of local phase transformations, the electron-positron field must be coupled to a new field, known as a gauge field. The exact form of the coupling and the equation satisfied by the gauge field are not uniquely prescribed by the invariance requirement, but there is a 'minimal' or simplest choice for which the invariance requirement is satisfied. The resulting 'minimal' gauge field for the electron-positron field is just the Maxwellian photon field. In this sense gauge invariance actually demands (explains) the existence of photons. This idea has been extended to cover the force fields associated with the so-called weak and strong interactions of the elementary particles[28]. As a result a new distinction grew up between 'matter' fields and 'force' fields. The matter fields were fermion fields associated with the so-called quarks and leptons. The force fields were the gauge fields derived by imposing invariance under appropriate generalizations of the local phase transformations we described above for QED, and these fields were boson fields. So we had the situation: the fermion fields in Nature described matter and the boson fields described their interactions[29]. In passing we note that this distinction between matter and force depended on accepting the arguments critized in Section 3 above that the quanta associated with the gauge fields were not really 'particles' at all. But then a new development, supersymmetry, blurred the distinction again[30]. This involves a symmetry transformation which exchanges bosons and fermions, and when expressed as a gauge principle leads to a theory of gravitational interaction known as supergravity. Combining again with the gauge symmetries characterising the unification of the weak, electromagnetic and

strong interactions leads to so-called extended supergravity which proposes a unification of all the known interactions and a unification of matter and force. What we want to do now is to comment briefly on the meaning of the term 'unification' in the scheme we have very briefly sketched above. The idea behind unifying two sorts of physical phenomena is that there is in fact just one kind of entity that enters into the unifying theory, where previously the phenomena were explained in terms of distinct independent entities. An example would be the unification of optical and electromagnetic phenomena achieved by 'identifying' the optical and electromagnetic ethers. A rather different case is the unification of electrical and magnetic phenomena as examplified in Maxwell's equations. Here the two vector fields E and H are combined into one entity, the electromagnetic field tensor. The reason why it is not appropriate to regard E and H as two entities is that under a passive transformation to a new space-time coordinate system, specified by the usual Lorentz tranformation, the distinction between the E and H field has no invariant significance. The invariant object is the 6-component antisymmetric field tensor itself. But the unification achieved in the gauge theories, although often assimilated to our second sort of example, is really somewhat different. Different kinds of particle are now regarded as different states of a single entity, these different states being transformed into one another under suitable gauge transformations. The classic example of such a device is to regard the neutron and proton as two states of a single entity, the nucleon. But this talk of a single entity with different states is really just a façon de parler. All that we can actually say is that if the gauge symmetry (local isospin rotations) were exact then physics could not decide for us which particle was a proton and which a neutron, but this is not the same thing as saying neutrons are protons, nor does it compel us to the view that neutrons and protons are just two states of a single entity (the nucleon). The distinction from the electric/magnetic example lies in the fact that the isospin transformations do not have an obvious passive interpretation. They are essentially active symmetry transformations which change one physical system into another

rather than a mere redescription of a single physical system as in passive symmetry transformations[31]. The, probably controversial, upshot of our argument is that the unification achieved in gauge theories is not obviously related to ontological economy. My point here is that the question of whether there are many natural kinds of elementary particles is not settled by arguments based on gauge symmetries. That does not mean that the notion 'natural kind' is not itself philosophically controversial.

7. The Problem of Individuality - Paraparticles and Parafields

The claim is often made that elementary particles are non-individuals in the sense that they do not possess what we called TI in Section 1. This is taken as a fundamental argument for using a field approach to the elementary particles[32]. This claim is usually based on a consideration of the quantum statistics of bosons and fermions. If we consider the possible states available to two indistinguishable particles distributed among two distinct 1-particle states, the possible arrangements in classical statistical mechanics are as follows:

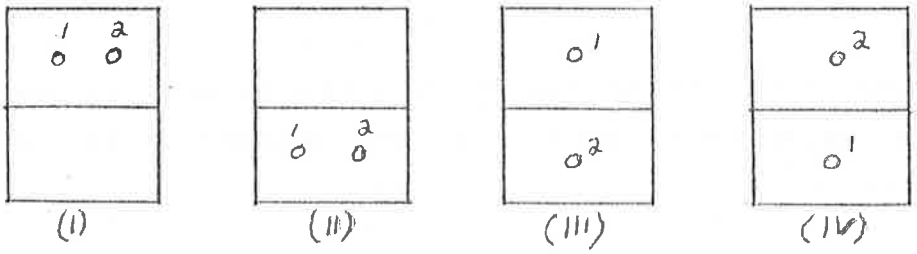

Fig 5.

The two boxes represent the two states and the labels 1 and 2 refer to the two particles (assuming TI). Each of these arrangements are given the same statistical weight. In quantum statistical mechanics the arrangements (III) and (IV) are regarded as one and the same state of the 2-particle system. This is taken as showing that the particles cannot possess TI, and hence are non-individuals.

There are a number of comments I would like to make about this argument. Trivially it cannot be conclusive, since it is perfectly possible to give an account of quantum statistical

mechanics using the first-quantized version of the N-particle problem in which labels (proper names) are ascribed to the particles. Consider the problem of distributing the two particles labelled 1 and 2 between two states specified by orthogonal wave functions $\psi_a(\underline{x})$ and $\psi_b(\underline{x})$.
There are four possible product wave functions

$$\psi_a(\underline{x}_1)\cdot\psi_a(\underline{x}_2) \qquad \text{I}$$

$$\psi_b(\underline{x}_1)\cdot\psi_b(\underline{x}_2) \qquad \text{II}$$

$$\psi_a(\underline{x}_1)\cdot\psi_b(\underline{x}_2) \qquad \text{III}$$

$$\psi_b(\underline{x}_1)\cdot\psi_a(\underline{x}_2) \qquad \text{IV}$$

which correspond to the four possibilities illustrated schematically in Fig.5. These wave functions are all orthogonal and span a 4-dimensional vector space. This space is equally well spanned by another set of orthogonal wave functions

$$\psi_a(\underline{x}_1)\cdot\psi_a(\underline{x}_2) \qquad \text{V}$$

$$\psi_b(\underline{x}_1)\cdot\psi_b(\underline{x}_2) \qquad \text{VI}$$

$$\frac{1}{\sqrt{2}}\left(\psi_a(\underline{x}_1)\cdot\psi_b(\underline{x}_2)+\psi_b(\underline{x}_1)\cdot\psi_a(\underline{x}_2)\right) \qquad \text{VII}$$

$$\frac{1}{\sqrt{2}}\left(\psi_a(\underline{x}_1)\cdot\psi_b(\underline{x}_2)-\psi_b(\underline{x}_1)\cdot\psi_a(\underline{x}_2)\right) \qquad \text{VIII}$$

The wave functions V and VI are identical with I and II and are symmetric under exchange of the particle labels 1 and 2. However the new linear combinations VII and VIII formed from III and IV are chosen to retain simple symmetry properties under the label permutation: VII is symmetric like V, VI, while VIII is antisymmetric (changes sign).

Now it is easy to show that if the Hamiltonian governing the time-development of the state vector is itsel symmetric under a permuation of particle labels, then a symmetric state will remain always symmetric and an antisymmetric state always antisymmetric. In other words if we impose the initial condition

that the state of the system is either symmetric or antisymmetric, then only one of the states VII or VIII is ever available to the system, and this explains why the statistical weight attaching to the pair of states VII and VIII gets halved. Quantum mechanics does not say VII and VIII are the same state anymore than III and IV, just that there is a dynamical restriction on the accessibility of certain states which is reflected in the attribution of statistical weights. This restriction arises from the symmetry of the Hamiltonian under label permutations. But this in turn can be deduced as a necessary condition for the indistinguishability of the particles. Firstly we explain what we mean by indistinguishability.

> The Indistinguishability Principle (IP): In quantum mechanics two particles are said to be indistinguishable if for any observable Q the expectation value of Q in any state $|\phi\rangle$ is the same as the expectation value of Q in the state $P|\phi\rangle$ where P is any permutation operator acting on the particle labels occurring in the specification of $|\phi\rangle$.

In other words no observable can be used to distinguish states which differ only in a permutation of particle labels. From this principle it follows as a necessary condition for indistinguishability that every observable, and in particular the Hamiltonian, must be a symmetric function of the particle labels. IP is here interpreted as a restriction not on the states of a quantum-mechanical system, but on the possible observables. It is also true but irrelevant that if IP is interpreted as a restriction on states, then a sufficient condition for its satisfaction is for $|\phi\rangle$ to be symmetric or antisymmetric under the action of P whatever the symmetry properties of Q[33]. Confusion over the question of whether IP restricts states or observables led to the widespread belief that one could deduce the alternative of boson or fermion character of a system of particles from their indistinguishability. The matter was only straightened out in the classic work of Messiah and Greenberg (1965), who showed that boson or fermion statistics were only two of a spectrum of possibilities, collectively known

as parastatistics. Particles obeying these statistics are known as paraparticles. Stolt and Taylor (1970a) classified paraparticles into three types: parabosons of finite order, parafermions of finite order and paraparticles of infinite order. Furthermore in their (1970b) Stolt and Taylor demonstrated the formal equivalence of parabosons and parafermions of finite order and Green's parafields discussed in Section 3. Stolt and Taylor conjecture that further generalization of the commutation relations for creation and annihilation operators could lead to a second-quantized version of the theory of paraparticles of infinite order[34]. So the quantum mechanics of indistinguishable particles can (probably) be given a formulation in terms of field excitations, but only an extreme form of positivism would hold that because we cannot tell which label attaches to which particle, therefore we are compelled to give up a description which does ascribe labels to particles.

We have already had occasion to refer to the possibility of a field description in classical mechanics. (See Sections 2 and 4 above). But how is this possible in the light of the Boltzmann counting appropriate to the classical statistical mechanics of indistinguishable particles which does treat the arrangements (III) and (IV) in Fig.5 as distinct? The answer to this question lies in the fact that the 'spike' fields which represent the particles in the field approach move around the phase space in continuous trajectories. This spatio-temporal continuity in trajectory can be used as a substitute for the non-available TI in attaching labels to the spikes. Two spikes have the same label if they both lie on a continuous phase space trajectory. This procedure enables us to distinguish the arrangements in which two spikes exchange location in phase space in terms of the histories (trajectories) of the spikes leading up to the two arrangements. This method of individuating field configurations is not in general available in QFT due to the overlapping wave functions of the 1-particle states. For very dilute gases, where overlap can be ignored, quantum statistics anyway degenerates into classical statistics.

8. The Vacuum and Virtual Particles

So far we have stressed the thesis of underdetermination as between field and particle approaches to QFT. Consider however the vacuum state, $|\Phi_0\rangle$, defined as the state where no particles are present, i.e. the state where the excitation numbers for all the modes are zero. We might expect, from the particle point of view, that $|\Phi_0\rangle$ is a state of non-being, of complete 'quiescence'. But in QFT this would be quite misleading. This is because QFT directs attention to the field amplitude, or for non-Hermitean fields, to local observables like energy density and these quantities, not being diagonal in the particle representation, exhibit fluctuations, even in the state $|\Phi_0\rangle$ [35]. These are known as vacuum fluctuations and are responsible for the zero-point energy 'discarded' in Section 2 above, and also serve to explain, in crude terms, self-energy effects like the Lamb shift and the anomalous magnetic moment of the electron in QED[36]. Perhaps the most striking demonstration of vacuum fluctuation is the so-called Casimir effect[37]. Here one computes the zero-point energy for the electromagnetic field in the presence of two uncharged conducting plates. This turns out to be a function of the distance between the plates, which demonstrates in fact an attractive force between them. Since one is working always with the vacuum state, there does not seem to be any room for a particle interpretation of what is going on. But this is only true if by a particle interpretation we mean an interpretation which only makes reference to quantities which are diagonal in the particle representation. This rules out by definition quantities like local field observables which create or destroy particles. But in the context of an extended particle interpretation which allows the possibility of particle creation and annihilation via the Fock space machinery, we can still talk of a particle interpretation of vacuum fluctuations.

Another aspect of the vacuum presents itself when we consider interacting fields. If we denote by H' the interaction Hamiltonian coupling two fields such as the electron-positron field and the electromagnetic field in QED, and by H_0 the free Hamiltonian of the uncoupled fields, then one must distinguish

the vacuum $|\Phi_0'\rangle$ of the coupled fields, corresponding to the lowest energy eigenstate of H_0+H' from $|\Phi_0\rangle$, the vacuum of the uncoupled fields which is the lowest energy eigenstate of H_0 alone. Indeed $|\Phi_0'\rangle$ can be formally expanded as a superposition of $|\Phi_0\rangle$ and appropriate many-particle eigenstates of H_0. This is often expressed by saying that $|\Phi_0'\rangle$ is a state containing no real particles but only virtual particles arising from the expansion of $|\Phi_0'\rangle$ in terms of eigenstates of H_0 as explained above. So the physical vacuum of the interacting fields involves the 'presence' of virtual particles. In terms of the Feynman diagrams the virtual particles correspond to the internal lines of the diagrams, the external lines corresponding to the real (bare) states. These lines, in the momentum representation of the Feynman diagram, carry 4-momentum variables which are conserved at each vertex, but in general do not lie on the mass-shell specified by $p_4^2 - (p_1^2+p_2^2+p_3^2)c^2 = m^2c^4$, where m is the rest-mass of the particle and c the velocity of light. This is why these particles are called virtual particles instead of the real particles corresponding to the external lines of Feynman diagrams which do lie on the mass-shell. This distinction replaces that appropriate to the older non-covariant perturbation techniques in which the virtual particles were on the mass-shell, but did not conserve energy in their creation and annihilation. Care should be taken not to read into the description in terms of virtual particles anything more 'concrete' than is warranted by the mathematics of solving a problem specified by the exact Hamiltonian H_0+H' in term of solutions of the problem specified by H_0 alone. In particular virtual particles have no direct connection with vacuum fluctuations as such, since these exist in the absence of field interactions.

Another confusion arises when describing the interaction between matter particles as due to the exchange of virtual particles associated with a force field. Students often wonder why such interactions can ever be anything but repulsive, on the model of two tennis players repelling each other by exchanging tennis balls! The explanation of this conundrum is that

conservation of momentum is only appropriate to Feynman diagrams in momentum space. But here there is no spatial location to tell us whether we are dealing with attraction or repulsion.

Fig 6.

In the two collisions illustrated in Fig. 6 we only know which involves attraction and which repulsion if we know on which side the particles pass each other. Merely knowing the momenta of the particles does not provide this information[38].

9. Conclusion

Throughout this paper we have looked at two approaches to QFT, the field and the particle approach. In the particle approach we are dealing with continuants, metaphysical entities which are reidentified through time in virtue of TI. To take account of the creation and annihilation of particles, the notion of continuant has to be modified somewhat from the classical conception of entities which persist through all time, continuants now persist only through limited intervals of time. But once the possibility of creation and annihilation of continuants is entertained then the description of change in terms of a continuous process of annihilation and recreation leads to an effective denial of the 'continuant' notion. The alternative description in terms of field configurations may be thought of in terms of a new category of entities we suggest calling ephemerals. These are entities which can be distinguished one from another at any given instant of time, but unlike continuants cannot be reidentified as the same entity in virtue of TI at different times, such reidentifications only being possible if notions of spatio-temporal continuity can be applied.

A moving field configuration or 'wave' in classical physics would be an example of a reidentifiable ephemeral. But ephemerals, entities like field configurations, can be created or destroyed without the somewhat awkward notion of continuants being 'started' and 'stopped' having to be invoked. Ephemerals seem natural candidates for the metaphysical category to which elementary particles should be assigned. This does not mean that each particle in a collection of particles corresponds in general to an ephemeral in a collection of ephemerals. A collection of indistinguishable particles is itself a single ephemeral. This is the sense in which the elementary particles can be regarded as bereft of individuality. The notion of particle as labelled individual can at all events only be retained if we are prepared to admit TI. This is something which modern philosophy has come to regard with a great deal of suspicion. We have employed the notion freely in this paper simply because there is no other way of making sense of the particle approach to the quantum mechanics of bosons and fermions[39]. It is often assumed that this is only true if we consider cases such as two bosons both in the same 1-particle state. But it should be noted that two bosons or two fermions distributed among two distinct 1-particle states partake with equal probabilistic weight of each state in the superposition demanded by symmetrization or antisymmetrization[40]. So in this sense the two particles always are in the same state. We do not want to take sides in this paper on the question whether TI is metaphysically admissible. We just want to note that any philosophical arguments against TI will serve to undermine the particle as opposed to the field approach in the theory of elementary particles. N.B. But even if from a purely formal point of view we argue for a thesis of underdetermination as between field and particle approaches we want to stress that this does no justice at all to the heuristic value of field theory. The emphasis in field theory is on the local quantities, field amplitudes, energy densities and so on. These quantities look very artificial from the particle point of view, not being diagonal in the particle representation of QFT. The history of

developments in QFT has been one of taking the field programme seriously and letting it lead on in directions which are 'natural' within that programme. So this is an interesting situation in which extralogical arguments may be used to resolve a situation of formal underdetermination.

Although classical wave configurations are examples of ephemerals this does not mean that the elementary 'particles' are just waves. If we accept the arguments that they do not possess TI, then the substantive aspect of particles is lacking for the elementary 'particles' in the philosophical sense of subtantial. But like the Cheshire cat , although the substantial particularity has gone, there remains a particle 'grin'. The elementary 'particles' are not particles but they are also not classical-type fields. They are quantum fields - ephemerals with a particle grin. And if you want me to stick my neck out, this is the ultimate nature of reality according to modern theoretical physics.

It should be noted that there was a very ambivalent attitude to QFT during the early development in the 1930 s . This was due to the occurrence of 'infinities' whenever calculations were pushed beyond the lowest non-vanishing order of perturbation theory. This suggested that there must be something wrong with QFT! The problem of the infinites was patched up by the renormalization theory of the late 1940 s [41], but the difficulty of not being able to calculate anything brought the programme to a halt so far as the strong interactions were concerned, while the non-renormalizability of weak interaction theory left QED alone as a successful (very successful) example of what QFT could achieve. With the development of gauge theories both these difficulties have been removed, and we now have renormalizable theories of strong, weak and electromagnetic interactions, in all of which precise calculations are possible[42]. The moral from the point of view of the Methodology of Scientific Programmes is never to abandon the programme too hastily either in the face of downright inconsistency (the infinities) or when the 'computation gap'[43] prevents the programme from making novel predictions and so renders the detailed articulation of the programme pragmatically immune to refutation in a sense that is all too familiar to theoretical physicists, although not perhaps sufficiently recognized, hitherto, by philosophers of science.

In a sense nothing has happened in QFT since around 1930. In Kuhnian terms there has been no revolution, no paradigm switch[44], just a gradual working out of the immense and really unsuspected technical richness of the theory. But in respect of the interpretation of QFT,

what the formalism really commits us to ontologically speaking, there has been too much rhetoric and too little informed and critical discussion. The object of this essay has been to entice (and assist) philosophers who would like to join in the arguments.

FOOTNOTES

1. Field theories of matter are thus more appropriate in the context of absolute theories of space-time. In relational theories the individuation of space-time points is itself dependent on the individuation of material objects. But unless we accept the controversial principle of the Identity of Indiscernibles, we can never individuate objects merely by enumerating their properties, so we are back to something like TI for the material objects. The escape from this resurgence of TI is usually argued for in terms of some privileged set of primary particulars linked in some way to the unitary nature of human consciousness. Compare, for example, Wiggins (1963). At all events, the idea of space-time points as individuals which have properties is in need of considerable clarification.

2. The most famous counterexample to this claim is, surprisingly, Boltzmann. Despite having effectively introduced transcendental individuals in his paper on statistical mechanics (Boltzmann (1877)), in his treatise on Mechanics (Boltzmann (1897)) he stresses spatio-temporal continuity of trajectory as the source of particle individuation.

3. See the translation of Newton's 'De Gravitatione et Aequipondio Fluidorum' in Hall and Hall (1962), especially pp. 138-140. Further discussion is given in Stein (1970).

4. The terminology is due to Newton-Smith (1981).

5. See Heimann and McGuire (1971) or McGuire (1974) for more detailed discussion.

6. See Faraday (1844).

7. Cf. Heilbron (1979).

8. For discussion of ether theories see in particular Doran (1975), Whittaker (1951) and Schaffner (1972).

9. See Silliman (1963) for discussion and references to the original papers.

10. Larmor's work is given particular emphasis in Doran (1975).

11. McCormmach (1970) is a useful reference here.

12. Cf. Elkana (1974) and Theobald (1966).

13. A good introduction to these ideas can be found in Misner (1974).

14. A standard reference for further details is Wentzel (1949).

15. For the harmonic oscillator p_k and q_k are themselves Hermitian operators. By a simple canonical transformation (from travelling to standing waves) the Hamiltonian (10) can be epxressed as a summation over 'real' oscillators. Compare Fermi (1932). The advantage of our approach is that it leads to quanta of definite momenta.

16. Such a distinction has been argued for in particular by Audi (1973). A reply to Audi's own arguments is given in Redhead (1977).

17. See Wightman (1972) for further discussion of this point.

18. Cf. Joos (1962) and Weinberg (1964) and (1965). Halpern (1968) provides a useful introduction to these ideas.

19. The local observables are actually averages of Q(x) defined over small space-time regions. Technically Q(x) itself in general generates states of infinite norm, so its range is not confined to the relevant Hilbert space. We shall ignore these refinements in our simplified presentation. See Dyson (1951) for further comments and Bohr and Rosenfeld ((1933) and (1950)) for the classic discussion of the measurability of fields.

20. For further discussion of this point and the contrast with nonlocality involved in the Einstein-Podolsky-Rosen paradox see Redhead (1982).

21. We are concerned with formal possibilities allowed by the theory. Briefly quarks were interpreted as parafermions of order 3. See Greenberg (1964).

22. Compare also Bromberg (1976) for further discussion.

23. For a discussion of the formal properties of the Liouville operator see Prigogine (1962).

24. The classic account of this doctrine is given by Maimonides (1904).

25. Dirac (1927) p.245. Comments on this paper have also been given by Bromberg (1977).

26. In cases where the field operator is non-Hermitian , as for the *non*relativistic Schrödinger field, local observables are still available to provide the wave picture.

27. A matter of fundamental importance here is the question of whether both fields must be quantized. Discussion of this point is provided by Brown and Redhead (1981).

28. A suitable non-technical introduction to the modern physics of elementary particles is provided by Mulvey (1981). At a much more detailed, although still non-technical, level, Pickering (1981) is an outstanding resource.

29. This simple scheme overlooks the role of the so-called Higg's fields used to break the symmetry of the vacuum and so allow massive gauge particles to appear in the theory. It is not entirely clear, however, that the Higg's mechanism for symmetry-breaking is not an ad hoc device, which will be eliminated in further fundamental developments.

30. See Freedman and van Nieuwenhuizen(1978) for a popular account. Technical detail is given in van Nieuwenhuizen (1981).

31. For further discussion of these matters see Redhead (1975).

32. See for example Post (1963). Further discussion on the question of individuality is given by Cortes (1976), Barnette (1978), Ginsberg (1981) and Teller (1982). Note that Cortes uses 'non-individual' in quite a different sense, distinct entities which are indistinguishable. But this really only makes sense if non-individuals do possess TI.

33. More formally the IP can be expressed as

$$\langle P\phi|Q|P\phi\rangle = \langle\phi|Q|\phi\rangle \quad \forall\phi, \forall Q$$

where $|P\phi\rangle$ is a convenient abbreviation for the state $P|\phi\rangle$.

But $\langle P\phi|Q|P\phi\rangle = \langle\phi|P^{-1}QP|\phi\rangle$

since P is a unitary operator. So considered as a restriction on observables it can easily be shown that IP implies $P^{-1}QP = Q$, i.e. $QP = PQ$, so P commutes with Q and hence Q must be a symmetric function of the particle labels. But considered as a restriction on states IP will be satisfied for any Q, symmetric or not, provided $|P\phi\rangle = \pm|\phi\rangle$.

34. Stolt and Taylor (1970) p.16.

35. In view of the assimilation of quantum fields to an assemblage of harmonic oscillators the existence of these fluctuations corresponds to the fact that in the quantum mechanics of the harmonic oscillator (assumed 1-dimensional) the ground-state wave function $\psi(q) = (\omega/\pi\hbar)^{1/4} e^{-\omega q^2/2\hbar}$ is an eigenstate of the energy $H = \frac{1}{2}(p^2 + \omega^2 q^2)$, where ω is the angular frequency of the oscillator, but this is not an eigenstate of the amplitude coordinate q. While $\langle q\rangle = 0$ for the ground state, it is easily shown that

$$\langle q^2\rangle = \int_{-\infty}^{+\infty} \psi^*(q)\, q^2\, \psi(q)\, dq = \hbar/2\omega$$

The standard deviation σ which measures the fluctuation in q is given by $\sigma = \sqrt{\langle q^2\rangle - (\langle q\rangle)^2} = \sqrt{\hbar/2\omega}$.
So the fluctuation is clearly non-vanishing in the ground-state.

36. See Welton (1948) and Koba (1940).

37. Casimir (1948). The effect was experimentally verified by Sparnaay (1958).

38. A simple argument shows that if the scattering angle is well-defined the transverse relative location must be uncertain by an amount large compared with the range of force mediated by the exchange particle. Thus we require $\Delta\theta \ll mc/p$, the typical relativistic scattering angle, where $\Delta\theta$ is the uncertainty in the scattering angle, m the mass of the exchanged particle and p the momentum of the incoming particles. But also $\Delta\theta \sim \hbar/p.d$ where d is the accuracy of transverse relative location of the particles. Hence we obtain $d \gg \hbar/mc$ as the condition for a well-defined scattering angle. But $\hbar/mc$ is the range of the force mediated by the exchange particle (cp. for example Redhead (1980) p.283).

39. Remember we are assuming throughout this paper the completeness of the quantum-mechanical description. If we allow the positions of individual particles, for example, as hidden parameters, the necessity of TI no longer follows in the particle approach.

40. This is not true with higher-order paraparticles but it is still true of course that due to the restriction to symmetric observables there is no way of telling which particle is in which state.

41. These developments are explained in Redhead (1980a). See also Cushing (1981). The history of QFT is also dealt with, rather sketchily, by Wentzel (1960), Weinberg (1977) and Weisskopf (1981).

42. In the case of strong interactions this is still not true, unfortunately, for 'long range' effects.

43. For a discussion of the computation gap, which occurs when a theory is too complicated to work with in producing precise empirical predictions, see Redhead (1980b).

44. But compare the development of the rival S-Matrix programme in the 1960 s. See Redhead (1980a) and Cushing (1982) for details.

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